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DIAGNOSING PUPIL PERFORMANCE IN DECIMAL MULTIPLICATION: COMPUTATION AND ESTIMATION

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Diagnosing Pupil Performance in Decimal Multiplication: Computation and Estimation

by

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University of Victoria

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Diagnosing Pupil Performance in Decimal Multiplication: Computation and Estimation*

Purpose

The study was designed to assess the ability of students in Grades 6, 7 and 8 to compute and estimate decimal products. Procedures and thinking patterns used by students and the types of errors made in decimal multiplication exercises were also investigated. A major objective of the project was the development of an instructional sequence and materials for teaching students the skills needed to estimate and check answers in multiplication situations.

Background and Related Research

Decimal arithmetic is receiving increased attention in elementary and middle school mathematics programs. The availability and power of the handheld calculator and the adoption of the metric system are two of the factors that have led to a reexamination of the teaching of decimals. Questions regarding both the placement of topics and the emphasis to be given the various skills are currently under consideration.

While it is generally agreed that students should have reasonable facility in using the standard paper and pencil algorithm for the four operations with decimals, it is recognized that complicated computations will often be done with a calculator. Developing the ability to estimate answers and check for reasonableness of results are considered major objectives of today's school mathematics curriculum (Johnson, 1979).

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Carpenter and others (1976) state:

> ... students need to develop some sort of quantitative intuition that allows them to compare numbers, figure out whether an answer makes sense, have some feel for the quantities represented by large numbers, and so on. (p. 301)

Research results consistently indicate that students have difficulty understanding and applying decimal concepts. In a study on decimal arithmetic with pupils in Grades 6, 7 and 8, Bruckner (1928) determined that the major source of error was a lack of understanding of place value. The most common errors in multiplication exercises were the misplacement or omission of the decimal point. More than 50 years later, in a report of the Second National Assessment of Mathematics in the United States, Carpenter and others (1981) concluded that the main source of difficulty in decimal computation was lack of a firm understanding of the place-value system and its relationship to decimals. It was found that while almost half of the 13 year olds were successful in multiplying decimals and in choosing the correct placement of the decimal point in a given answer, where the decimal point was incorrectly placed, the answer was often unreasonable. The results of an estimation exercise further indicated that students had little idea of what number the product should be near.

In the B.C. Mathematics Assessment (Robitaille, 1981) items requiring the application of place-value concepts were rated as marginal or weak. An analysis of a multiplication item led the investigators to conclude that while students know the rule for placing the decimal point, they do not use estimation to check that the answer it produces is reasonable.

The Manitoba Assessment Program 1982 (Wells, 1983) recommended that the topic of place value be reviewed at the Grade 6 level and that "educators pay more attention to the learning of operations with decimals."

Procedure

Test

A paper and pencil test on multiplying decimals was constructed for the study. Part I of the test consisted of twelve computational exercises presented in horizontal form. In seven of these questions one factor was a power of ten or both factors contained only a single non-zero digit. Students were instructed to show any work they did in the space provided with each question. Part II consisted of ten multiple choice estimation items. Seven of these required students to identify the proper order of magnitude of the product and were parallel to the seven computation exercises previously described.

The test was administered to ten classes of students in Grades 6, 7 and 8 in a lower Vancouver Island middle school. A stratified random sample of 150 test papers was selected for analysis. In Grades 7 and 8 ten girls and ten boys were selected at each of three achievement levels - high, middle and low. At the Grade 6 level there were only five students in each cell, since some classes in the school had not yet introduced multiplication of decimals. Achievement levels were determined by report card grades in mathematics: high - A, middle - C+, low - C-.

Individual Interviews

Oral interviews were conducted with 25 students - eight each in Grades 6 and 7 and nine in Grade 8 - in the same school who did not take

the written test. During the interview which lasted about 15 minutes, questions were presented to the subject on cards, one item per card. Some questions asked the student to provide a model or example to illustrate a decimal concept. Several of the cards contained computation and estimation questions similar to those on the paper and pencil test. As the student worked the exercises, he or she was asked to describe out loud the procedure being used to obtain the answer and to explain why the rule was (mathematically) valid. Due to time constraints and differences due to grade level, the same number of items was not presented to all subjects interviewed.

Instructional Unit

Based on the results of the test and oral interviews and an examination of literature on the teaching of decimals and estimation, a five-lesson unit of instruction designed to develop the understanding and skills needed to estimate decimal products was developed. The unit was piloted in a Grade 7 class in a Victoria elementary school. Pre- and post-tests were used to evaluate student learning. The unit was then revised and extended.

Test Results

Performance by students in Grades 6, 7 and 8 on three of the computation exercises is given in Table 1. While the Grade 6 students did as well as the Grade 7's and 8's on the easy items, the older students scored higher on the more difficult exercises.

Table 1
Performance on Selected Computational Items at Three Grade Levels

Item	Percent Correct		
	Grade 6	Grade 7	Grade 8
0.5 x 24	90	88	85
7.29 x 0.01	57	85	83
0.42 x 0.617	53	62	77

Table 2 summarizes the results on the computation and estimation subtests by students grouped according to grade and achievement level.

Table 2
Performance on the Computation and Estimation Subtests
by Grade and Achievement Levels

Achievement Level	Percent Correct							
	Grade 6		Grade 7		Grade 8		Total	
	Comp.	Est.	Comp.	Est.	Comp.	Est.	Comp.	Est.
High	80	44	91	58	90	61	88	56
Middle	61	25	79	38	88	50	79	40
Low	58	24	67	28	69	39	66	31
Total	67	31	79	41	82	50	78	43

Students did much better on the computation exercises than on the estimation items. Performance on both subtests increased with grade level and achievement level, as would be expected.

Test results for girls and boys at the three achievement levels are given in Table 3.

Table 3
Performance on Computation and Estimation Subtests
by Achievement Level and Sex

Achievement Level	Percent Correct			
	Girls		Boys	
	Comp.	Est.	Comp.	Est.
High	87	49	90	64
Middle	77	36	81	44
Low	72	34	60	28
Total	79	40	77	46

Overall there was no difference in achievement between the girls and boys in computation, but in estimation the boys scored slightly higher than the girls ($p = .067$). Statistically significant sex by achievement level interactions were indicated for both subtests. It can be observed that at the low achievement level girls tended to score higher than boys, but at the two higher achievement levels the situation was reversed.

As noted earlier, students were much more successful in computing decimal products than in estimating them. For example, while 77% of the total group obtained the correct answer for 0.4 x 0.2, on the related estimation item, 0.378 x 0.24, only 37% chose the alternative with the decimal point correctly positioned relative to the digit 8 (Table 4).

Table 4
Estimation Item No. 5

5.		Percent of Students	
	0.00008	21	
0.378 x 0.24	0.008	20	
	0.08	37	*
	0.8	23	

The performance of students on the seven related pairs of computation and estimation items is provided in Table 5.

Table 5
Performance on Related Computation and Estimation Items

Computation Item	Percent Correct	Estimation Item	Percent Correct
6.14 x 10	87	6.138 x 10.35	86
50 x 600	85	48.92 x 623.8	64
7.29 x 0.01	79	6.825 x 0.012	48
7000 x 0.002	79	7189 x 0.0023	46
0.06 x 0.009	72	0.062 x 0.0089	44
1000 x 0.04	78	998.6 x 0.0428	39
0.4 x 0.2	77	0.378 x 0.24	37
Total	80%		52%

The estimation item which proved to be the most difficult for the students required them to place the decimal point in the answer to a multiplication exercise (Table 6).

Table 6
Estimation Item No. 8

8. The decimal point has not yet been placed in the answer to the exercise below.

$$358.6 \times 0.25 = \boxed{8\ 9\ 6\ 5}$$

Which shows the correct placement?

	Percent of Students	
0.8965	2	
896.5	6	
8.965	80	
89.65	11	*

The number of students choosing the correct answer was less than what could be expected by chance, while 80% of the students selected the answer with three decimal places. Even though the item was part of the estimation section of the test, students simply applied the rule for placing the decimal point in a product and did not use estimation to check that the answer was reasonable. On the corresponding computation item, 360 x 0.25, 81% of the students obtained the correct answer. An examination of the 28 incorrect answers revealed that the decimal point was misplaced by only seven of the students.

The last two estimation items were answered correctly by only about one-quarter of the students (Table 7).

Table 7
Estimation Items No. 9 and No. 10

9. The four exercises have approximately the same answer. One of the exercises has as its exact answer the number below.

2401.5473

Find which exercise has this answer by using estimation to eliminate the other three.

	Percent of Students	
59.77 x 38.59	19	
23.57 x 101.89	29	*
83.47 x 34.69	18	
240.87 x 10.09	33	

10. Lisa multiplied to find the answers for the four exercises, and then rounded them to the nearest dollar. Looking back on her work she noticed that one of her answers seemed unreasonable. Use estimation to find which answer is wrong.

	Percent of Students	
59 x $205.82 = $12 143	13	
192 x $287.69 = $55 236	32	
85 x $ 75.24 = $ 6 395	31	
801 x $510.95 = $40 927	23	*

Computational Procedures

Analysis of work shown on the computation subtest revealed that students had used a variety of procedures and algorithms for multiplying two numbers. The different computational forms were organized into three main categories. A short procedure was assumed to have been used when the answer was written next to the question given in horizontal format and the question had not been rewritten in a vertical format. Short algorithms for multiplying by a power of ten or finding the product of two multiples of products of ten with single non-zero digits involve "adding zeros" or "moving the decimal point." In 1000 x 0.04, one moves the decimal point in 0.04 three places to the right, or in 4000, two places to the left. An intermediate form was one in which the question had been rewritten in vertical format and an efficient algorithm used to obtain the answer. In algorithms in the long category, rows of zeros were shown as partial products or unnecessary zeros in one or both factors were multiplied.

The three types of procedures are illustrated below for the exercise 1000 x 0.04. The numbers in brackets indicate the frequency of use.

Short

40	0040.	40.00	40.0	
(16)	(7)	(5)	(9)	(37)

Intermediate

1000	0.04	
.04	1000	
40.00	40.00	
(40)	(9)	(49)

Long

1000	1000	1000.00	.04	0.04		
.04	0.04	0.04	1000	1000		
4000	4000	40.0000	00	000		
0000	00000		000	0000		
040.00	000000		0000	00000		
	0040.00		04000	004000		
			040.00	0040.00		
(18)	(33)	(2)	(1)	(8)	(64)	(148)

Examples of long computational forms for the other eleven items are illustrated below.

61.4 x 10	0.4 x 0.2	600 x 50	7.29 x 0.01
10.00	0.4	50	0.01
6.14	0.2	600	7.29
4000	08	00	009
10000	000	00	002
600000	0.08	300	007
61.4000		30000	0.0729

0.06 x 0.009	7000 x 0.02	0.81 x 7	24.0.5
0.06	7000	7.00	24
0.009	0.02	0.81	0.5
054	14000	700	120
0000	0000	5600	000
00000	0000	000	12.0
.00054	0140.00	5.6700	

360 x 0.25	3.92 x 4.5	0.617 x 0.42
360.00	3.92	0.617
000.25	4.50	0.420
180000	000	0000
720000	19600	1234
90.0000	156800	2468
	17.6400	0000
		0.259140

Table 8 gives the frequency of use of the three types of forms for the twelve items by the 150 students. Some items were not attempted by all students

Table 8
Use of Three Types of Computational Procedures

Item	Short	Intermediate	Long
61.4 x 10	41	32	77
1000 x 0.04	37	49	62
0.4 x 0.2	35	62	51
50 x 600	63	25	62
7.29 x 0.01	30	54	65
0.06 x 0.009	27	71	51
7000 x 0.002	30	70	45
	25%	35%	40%
7 x 0.81	12	130	7
0.5 x 24	14	67	68
4.5 x 3.92	0	117	28
360 x 0.25	6	99	44
0.42 x 0.617	0	108	34
	4%	71%	25%

For the first seven items listed long, intermediate and short forms were used in 25%, 35% and 40% of the exercises, respectively. For the last five items in which a short form as defined would normally not be used, an intermediate form was used 71% of the time and a long form 25%.

The frequency of use of a long form on the seven computational items with estimation item counterparts, by grade and achievement level, is given in Table 9.

Table 9
Use of a Long Computational Form on Seven Items
by Grade and Achievement Levels

Achievement Level	Percent of Exercises			
	Grade 6	Grade 7	Grade 8	Total
High	31	25	21	25
Middle	56	29	41	39
Low	59	63	47	56
Total	49	39	36	40

As indicated in the table the frequency of use of a long form decreases from Grade 6 to Grade 7 to Grade 8 and from the low to the middle to the high achievement level. On these seven items girls used a long form 49% of the time whereas boys used it only 32% of the time.

Table 10 summarizes the performance on the seven parallel compuation and estimation items according to the type of computational form used.

Table 10
Performance on Seven Parallel Computation and Estimation Items by Computational Form

	Percent Correct	
	Short/Intermediate	Long
Computation	80	80
Estimation	55	48
Estimation given correct computation	59	50

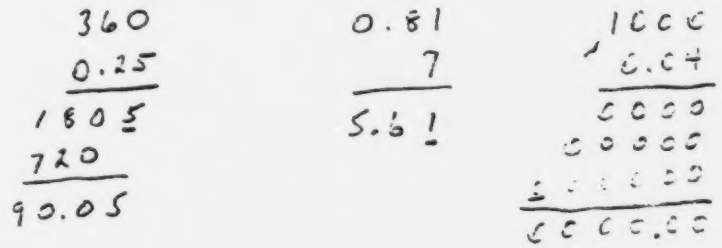
While the type of procedure used did not affect performance on the seven computation items, students who used a short or intermediate algorithm on a computation item achieved a higher success rate on the corresponding estimation item than students who used a long form. The difference between these two groups in estimation was even greater when answers obtained for the corresponding computation items were correct.

Error Categories

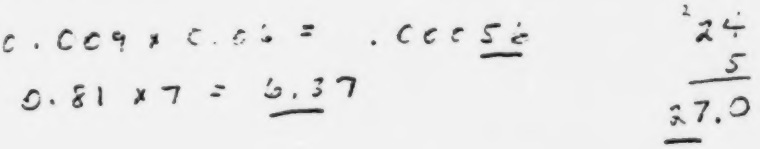
Errors made by students on the twelve computational questions were analyzed and classified. Seven main categories and several subcategories of errors were identified.

1. Multiplication fact errors

   (a) Zero - one facts



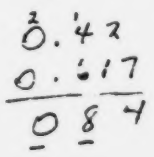
   (b) Other facts



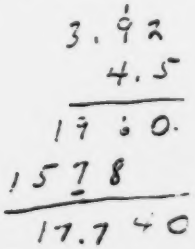
2. Carrying errors

   (a) Partial products

      (i) Carried digit not added



      (ii) Carried digit from previous row added



(iii) Digit carried to zero added to multiplier.

$$\begin{array}{r} 0.5 \\ \times\ 24 \\ \hline 60 \\ 100 \\ \hline 16.0 \end{array}$$

(iv) Carried digit multiplied

$$\begin{array}{r} 0.002 \\ \times\ 7000 \\ \hline 0074.000 \end{array}$$

(b) Column addition

$$\begin{array}{r} 3.92 \\ \times\ 4.5 \\ \hline 1960 \\ 1568 \\ \hline 16.\overline{}\overline{}40 \end{array}$$

3. Alignment errors

(a) Facts ending in zero

$$\begin{array}{r} 600 \\ \times\ 50 \\ \hline 000 \\ 3000 \\ \hline 3000 \end{array}$$

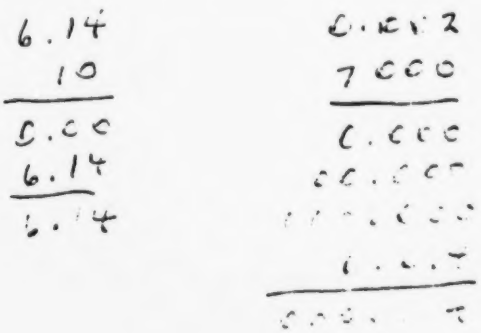
$$\begin{array}{r} 3.92 \\ \times\ 4.50 \\ \hline 000 \\ 19600 \\ 156800 \\ \hline 155.7600 \end{array}$$

(b) Carried number given place value position

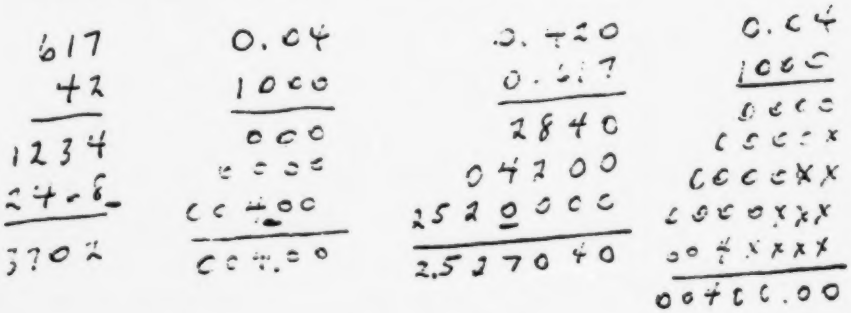
$$\begin{array}{r} .42 \\ \times\ .617 \\ \hline 2'94 \\ 42 \\ 25\overline{} \\ \hline 277\overline{} \end{array}$$

$$\begin{array}{r} .25 \\ \times\ 60 \\ \hline 1500 \\ 75\overline{}0 \\ \hline 2\overline{}.00 \end{array}$$

(c) Decimal point in partial product

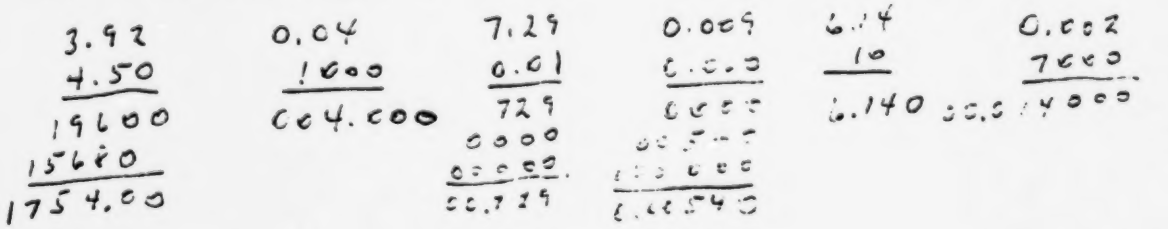


(d) Missing or extra zeros



4. Decimal point placement errors

(a) Incorrect counting rule



(same number as in factors) (count zeros in 1000) (count non zero dec. places) (don't count annexed zero) (add zero) (count dec. places and zeros)

(b) Incorrect application of counting rule

6.14 10 614.00	7.29 0.01 729 0000 00000 0072.9	0.06 0.009 .54000	7000 .002 14000.000
(add zeros)	(count from left)	(add zeros on right)	(count decimal places and add zeros)

(c) Decimal point omitted

6.14 10 000 6140 6140	0.4 0.2 08 80 088

5. Algorithm errors

(a) Discernible pattern

0.06 0.009 .549 000 0.549	.5 24 220	.617 .420 .02:340	.91 7 64	15000.000 .004 / 100.000.000

0.009 × 0.06 = 96.00

(b) Non-discernible pattern

7000 × 0.002 = 90.00

0.25
120
450
3700
4150

6. Numeral errors

(a) Question or answer copied incorrectly

1000 x 0.04

1000
00.4

3.92 x 4.5

3.95
4.5

(b) Digit poorly formed

0.009
0.06
0654
00000
00060
0.00114

0.617
.42
1234
24630
000000
.25834

340
.25
1800
7200
60000
6900.00

340
25
1800
7200
8500

7. Incomplete solutions

(a) Partial solution

7.29
0.01
729
000x
000xx

0.009
0.01
00-9
0

(b) No work or answer shown

Table 11 shows the number of errors in each category for each of the twelve items. In 24 exercises, two different types of errors were identified. Thus while 395 exercises were marked wrong, a total of 419 errors were categorized.

Forty-one percent of the errors involved misplacement or omission of the decimal point in the answer. This type of error was most common

in the seven exercises involving multiplying by a power of ten or two numbers with single non-zero digits. For example, while 25 students misplaced the decimal point in the answer to 7.29 x 0.01, only seven made this kind of error in calculating 4.5 x 3.92. There were, however, a total of 70 errors made on the latter item but only 32 on the former.

Table 11

Error Analysis by Item

Item	Error Category							
	Mult. Fact	Carrying	Align-ment	Dec. point placement	Algor-ithm	Numeral	Incom-plete	Total
.14 x 10	4		3	10	1		2	20
000 x 0.04	3		1	21	4	2	1	32
.4 x 0.2			2	27	1	1		31
0 x 600	1		14	6	1		1	23
.29 x 0.01			6	25			1	32
.06 x 0.009	8	1	3	24	2	1	1	40
300 x 0.002	2	1	2	17	2	1	4	29
x 0.81	13			12	3	5	1	34
.5 x 24	5	1	3	8	1	1		19
.5 x 3.92	11	31	6	7	6	2	7	70
50 x 0.25	4	7	6	7	4	2	2	32
.42 x 0.617	9	14	6	11	3	4	10	57
otal Number	60	55	52	175	28	19	30	419
ercent of Total	14%	13%	12%	41%	7%	6%	7%	100%
ouble errors"	15	14	5	5	4	5		48

Errors Involving Zero

Many of the errors within each of the categories involved zero, either as a number or as a place holder. Zero has always been a source of particular difficulty in computation. The use of unnecessary zeros in the algorithm provides increased opportunities for errors of various types. Teaching students to multiply without using the zero preceding the decimal point in numbers such as 0.4, and without writing in rows of zeros as partial products would help reduce the number of incorrect answers due to zero errors. Examples of various types of multiplication errors involving zero are now provided.

1. Multiplication facts involving zero

(a) $0 \times n = n$

$$\begin{array}{r} 0.4 \\ \underline{0.2} \\ 08 \\ 0\,\underline{4} \\ \hline 0.48 \end{array}$$

(b) $n \times 1 = 0$

$$\begin{array}{r} 7.00 \\ \underline{0.81} \\ \underline{0}.00 \\ 56\,00 \\ 00000 \\ \hline 0.5600 \end{array}$$

2. Carrying to zero

(a) carried digit not added

$$\begin{array}{r} 0.617 \\ \underline{0.42} \\ \underline{0}\ 234 \end{array}$$

(b) carried digit added to multiplier

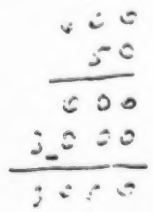
$$\begin{array}{r} {}^{2}0.5 \\ \underline{14} \\ \underline{6}\,0 \end{array}$$

(c) carried digit multiplied

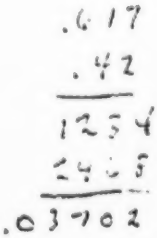
$$\begin{array}{r} 0.002 \\ \underline{7000} \\ 00\,\underline{7}\,4000 \end{array}$$

3. Alignment of partial products

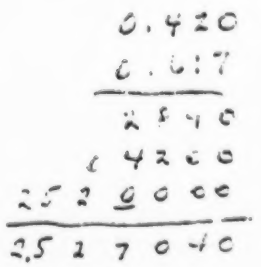
(a) Basic fact ends in zero



(b) Missing zeros

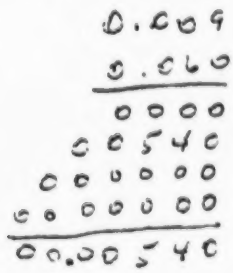


(c) Extra zeros

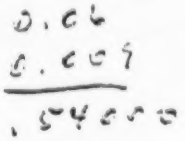


4. Placement of decimal point

(a) Incorrect counting rule



(b) Incorrect use of counting rule

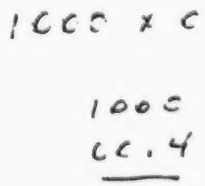


5. Defective algorithm

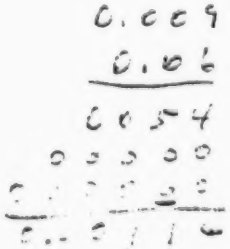
$$0.009 \times 0.06 = 96.00$$

6. Numeral mistakes

(a) Question incorrectly copied



(b) Digit poorly formed



Individual Interviews

Without exception the students interviewed were pleasant and cooperative, and responded positively to this opportunity to "talk mathematics."

Language and Notation

When asked to read 2.3, most students would say "two point three" or "two decimal three." Rarely, even when pressed, would a student say "two and three tenths." Grade 6 subjects were asked to use base ten blocks to model 2.3. Seven of the eight did this correctly.

The other individual arranged the blocks as illustrated below.

In response to the question "What does the decimal point mean?" students usually said that it separated the whole number and the fraction.

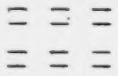
Although students had little difficulty identifying the place value of each digit in 0.627, no one, even with encouragement, would read the number as "six hundred twenty-seven thousandths."

Rules and Algorithms

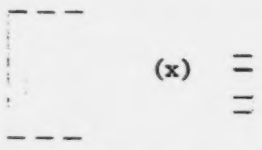
Thirteen of twenty students who were asked to choose the greater of 0.06 and 0.057 selected the correct number. Reasons given were "six hundredths is more than 5 hundredths" and "60 thousandths is more than 57 thousandths." One student said the two numbers were equal since .057 is rounded to 0.06. Sixteen of the twenty stated that 0.3, 0.30, and 0.300 were equal, but few could offer an explanation as to why zeros could be added or dropped after the 3. Only one student stated that three tenths equals thirty hundredths.

All subjects gave the correct answer to 3 x 0.2. The grade 6 pupils

were asked to model the number sentence using the blocks. Only one made three groups of two tenths.



The others arranged the blocks as follows and were unable to explain where the answer six tenths came from.



The Grade 7 and 8 subjects were asked to make up a story problem for 3 x 0.2. About half could suggest an appropriate situation. Some examples given were: the perimeter of a triangle with sides of length 0.2; 0.2 of a pencil, 3 pencils; one fifth of 3 apples; .

For the item 0.4 x 0.2, 18 of the 25 subjects gave the correct answer; the other seven gave the answer 0.8. Students who gave the correct answer were asked to justify (beyond stating the rule) why the answer was 0.08 instead of 0.8. Two subjects said that 0.8 would be the answer to 4 x 0.2, but no one related the question to place value or fractions (tenths x tenths = hundredths). Typical responses were: "that's just the way you do it"; "somebody made the rule up"; "I've never heard why." With respect to the question 4.5 x 3.92, all but one student could explain how to place the decimal point in the answer. After having stated the rule, three students then changed their answer to the previous question from 0.8 to 0.08.

Computational items such as 50 x 600, 1000 x 0.04 and 7.29 x 0.01 revealed students using short methods and long algorithms. Direct procedures involved mental arithmetic and rules for locating the decimal point. Use of the multiplication algorithm for this type of question is illustrated below.

$$
\begin{array}{r}
1000 \\
\times\ .04 \\
\hline
4000 \\
0\ 0\ 0\ 0\ 0 \\
\hline
040.00
\end{array}
$$

Students using a long procedure were asked if they thought the answer could be obtained in an easier or a shorter way or if there were rows of work that were unnecessary. They would often reply that while they remembered being shown a shorter method, they felt more secure doing all problems this way.

Estimation Strategies

When asked to estimate the answer to 6.138 x 10.35, most students gave 60 and said that they had rounded the numbers to 6 and 10.

For 48.52 x 623.8, many students rounded to the nearest whole number (49 x 624) or rounded both numbers to the nearest ten (50 x 620). Reluctance or inability to round to the leading digit was also displayed for 6.825 x 0.012. Common responses were 7 x 0 or 6.8 x 0 (producing an estimate of zero) and 6.83 x 0.01. Many students stated that both numbers were to be rounded to the same decimal place (which is appropriate in estimation in addition).

Eleven of 16 Grade 7 and 8 subjects arrived at a good estimate for 0.378 x 0.24. Eight of the good estimates were 0.4 x 0.2 = 0.08, one was 0.400 x 0.25 = 0.1, and one was $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100}$. Three students rounded to .400 x .20 but two made errors in computing the product. The incorrect answers were .8000 (8 + three zeros) and .80000 (8 + zeros to five decimal places).

Common strategies used in estimating the answer to 358.6 x 0.25 were 358.6 x 0.3, 359 x 0, and 360 x 0.3. No one suggested 360 x $\frac{1}{4}$. Before leaving this question, each student was given assistance as required in obtaining a reasonable estimate of the product, such as 108 or 120. The next card containing the following item was then presented.

Place the decimal point in the answer.

358.6 x 0.25 = 8 9 6 5

Invariably the student would count the decimal places in the two factors and give the answer 8.965. The subject would then be shown his or her estimate previously obtained for the same question and asked to again place the decimal point. Most students recognized the apparent contradiction. When pressed to decide which of the answers was correct they would usually place more confidence on the rule than on their estimate. The students would then be asked to do the problem on a calculator and consider the answer. Two Grade 8 students suggested that a trailing zero had been dropped. The other students remained puzzled.

In the estimation question 6.138 x 10.35, several students gave answers which indicated a lack of understanding of the multiplication algorithm.

60.56 (60 + .14 x .4)

60.39 (60 + .13 x .3)

60 + .138 x .35

In summary, the individual interviews provided some interesting insights regarding student understanding (and misunderstanding) of basic decimal concepts and revealed a variety of strategies for computing and estimating products.

Instructional Unit

Overview

An instructional unit on estimating decimal products was developed and taught by the investigator to a Grade 7 class at Fairburn Elementary School in Victoria. The class consisted of eleven girls and twenty boys exhibiting a wide range of achievement in mathematics. Instruction was given one hour a day for five days as follows.

Day 1. Place Value: Reading and writing multidigit numbers; identifying place value positions for whole numbers and decimals.

Day 2. Order and Rounding: Comparing and ordering whole numbers and decimals; rounding whole numbers and decimals to the leading non-zero digit.

Day 3. Multiplication by Powers of Ten: Multiplying whole numbers and decimals by 10, 100, 1000 and by 0.1, 0.01 and 0.001; finding the product of two powers of ten or two multiples of powers of ten.

Day 4. Estimating Products: Using numbers rounded to the leading digit.

Day 5. Recognizing Incorrect Answers: Identifying answers that are reasonable but too high or low, or with the wrong final digit.

Evaluation

A pre-test was administered during the week prior to the instruction and a post-test was given during the week following the teaching of the unit. Table 12 gives the results of the pre- and post-tests for the seven parallel computation and estimation items. These items reflect the main emphasis of the instruction given the class.

Table 12

Pre- and Post-Test Results for Computation and Estimation Items

Number Correct (n = 30)					
Computation			Estimation		
	Pre	Post		Pre	Post
6.14 x 10	30	30	6.138 x 10.35	24	26
50 x 60	30	29	48.92 x 623.8	24	23
0.4 x 0.2	26	27	0.378 x 0.24	9	19
1000 x 0.04	27	28	998.6 x 0.0428	14	15
0.06 x 0.009	25	30	0.062 x 0.0087	15	20
7000 x 0.002	26	26	7189 x 0.0023	18	21
7.29 x 0.01	27	30	6.825 x 0.012	15	24
	91%	95%		56%	70%

The students performed well on the computation items prior to the instruction and improved slightly on the post-test. The improvement on the estimation items is modest but encouraging. A fairly wide gap remained between students' ability to multiply by a power of ten or to multiply two numbers rounded to a leading digit and their ability to first create these numbers from multidigit numbers and then find the product. The preference displayed by some students to use the long algorithm to compute their answers appeared to be largely unaffected by the week's instruction.

The other three estimation items were more difficult, requiring sophisticated skills which were introduced only briefly on the last day of instruction. Pre- and post-test averages for these three items were 21% and 28%.

In general, while students did make some progress during the week in developing and improving their skill in estimating products, it is obvious that this goal needs to be tackled over a longer period of time, and that more practice and reinforcement of specific estimation strategies are required. The unit was subsequently revised and extended to provide for these concerns. The revised unit includes a sequence of skills, a lesson overview, worksheets and descriptions of materials, activities and games to promote understanding and provide practice.

References

Ashlock, Robert B. Introducing decimal fractions with the meterstick. Arithmetic Teacher, March 1976, 201-206.

Brueckner, Leo J. Analysis of difficulties in decimals. Elementary School Journal, 29, 1928, 32-41.

Carpenter, Thomas P. and others. Notes from National Assessment: Estimation. Arithmetic Teacher, April 1976, 296-302.

Carpenter, Thomas P. and others. Decimals: Results and implications from National Assessment. Arithmetic Teacher, April 1981, 34-37.

Grossman, Anne S. Decimal notation: An important research finding. Arithmetic Teacher, May 1983, 32-33.

Holmberg, Verda. Do Decimals First. Haywood, California: Activity Resources Company, Inc., 1976.

Johnson, David C. Teaching estimation and reasonableness of results. Arithmetic Teacher, September 1979, 34-35.

Kidder, Richard F. Ditton's dilemma, or What to do about decimals. Arithmetic Teacher, October 1980, 44-46.

Kilian, Lawrence and others. Errors that are common in multiplication. Arithmetic Teacher, January 1980, 22-25.

National Council of Teachers of Mathematics. Ideas from the Arithmetic Teacher, Middle School. Reston, Virginia: NCTM, 1982, 45.

Robitaille, David F. (Ed.) The 1981 B.C. Mathematics Assessment General Report. Victoria: Ministry of Education, 1981, 136-139.

Schmulz, Rosemary. A visual approach to decimals. Arithmetic Teacher, May 1978, 22-25.

Troutman, Andrea P. and Lichtenberg, Betty K. Mathematics: A good beginning - strategies for teaching children. Monterey, California: Brooks/Cole Publishing Company, 1982.

Victoria School District (#61) Teachers. Rounding and Estimation. 1982.

Victoria School District (#61) Teachers. Place Value. 1982.

Wells, Alan. Can Manitoba students add? A note on the Manitoba Mathematics Assessment Program. The Canadian Mathematics Teacher, 1983, 9.

Zawujewski, Judith. Initial decimal concepts: Are they really so easy? Arithmetic Teacher, March 1983, 52-56.

ESTIMATING DECIMAL PRODUCTS

Sequence of Skills

1. Place Value

   (a) Read and write a multidigit number

   The numeral 700 020 006.018 is read, "seven hundred million, twenty thousand, six and eighteen thousandths."

   (b) Identify place value in a multidigit number

   In 694 702.135, the 3 is in the hundredths place and means 0.03 ("three hundredths"). The 9 is in the ten thousands place and means 90 000 ("ninety thousand").

2. Order and Comparison

   (a) Continue counting sequences

   4.07, 4.08, 4.09, 4.10, 4.11

   (b) Identify equivalent decimal forms

   0.6 0.06 0.60 0.006 0.060 0.600

   (c) Find a number between two given numbers

   0.72 0.721 0.73

   (d) Compare two numbers

   0.7 > 0.68

   (e) Order a set of numbers from least to greatest

   3.461 3.61 3.64

3. Approximate and Rounded Numbers

   (a) Write the leading digit estimate for a number

   7964.913 → 7000
   0.0982 → 0.09

   (b) Write the rounded leading digit estimate for a number

   7964.913 → 8000
   0.0982 → 0.1

4. Multiplication by Powers of Ten

   (a) Multiply a number by 10, 100, 1000

   $1000 \times 3.2 = 3200$

   (b) Multiply a number by 0.1, 0.01, 0.001

   $8.46 \times 0.01 = 0.0846$

   (c) Find the product of two powers of ten

   $1000 \times 0.01 = 10$

   (d) Find the product of two multiples of powers of ten

   $40 \times 0.008 = 0.32$

5. Estimating Products

   (a) Use leading digit estimators

   $862 \times 0.35 \longrightarrow 800 \times 0.3 = 240$

   (b) Use rounded leading digit estimators

   $862 \times 0.35 \longrightarrow 900 \times 0.4 = 360$

   (c) Use "number sense" estimators

   $862 \times 0.35 \longrightarrow 900 \times 0.3 = 270$

   (d) Use fraction estimators

   $862 \times 0.35 \longrightarrow 900 \times \frac{1}{3} = 300$

6. Recognizing Incorrect Answers

   (a) Identify unreasonable answers

   $34 \times 62 = 218$ $\qquad$ $(30 \times 60 = 1800)$

   (b) Identify answers that are too high or too low

   $34 \times 62 = 1798$

   $34 \times 62 = 1808$

   (c) Identify answers with a wrong final digit

   $34 \times 62 = 2106$

   (d) Identify answers with a misplaced decimal point

   $13.5 \times 4.2 = 5.67$

Estimating Decimal Products

Unit Overview

Lesson 1. Place Value - Whole Numbers

Materials and Aids for Teaching Place Value sheet
Place Value Charts sheet
Worksheet #1

Lesson 2. Place Value - Decimals

Materials and Aids for Teaching Place Value sheet
Place Value Charts sheet
Worksheet #2

Lesson 3. Order and Comparison

Oral and Blackboard Activities sheet
Place Value Game sheet
Worksheet #3

Lesson 4. Approximate and Rounded Numbers

Worksheet #4

Lesson 5. Multiplying by Powers of Ten

Worksheet #5

Lesson 6. Products of Multiples of Powers of Ten

Worksheet #6

Lesson 7. Leading Digit Estimators

Play Estimating Decimal Products sheet
Worksheet #7

Lesson 8. Fraction Estimators

Worksheet #8

Lesson 9. Recognizing Incorrect Answers

Worksheet #9

Lesson 10. Checking Answers

The Product is Right sheet
Worksheet #10

Name _____

ESTIMATING PRODUCTS

Worksheet #1: Place Value (Whole Numbers)

1. Write each number as it is read to you.

   (a) _____ (469 703 120)

   (b) _____ (900 080 005)

2. Write in words the number 204 630 800

   _____

3. For the number 621 739 845

   (a) Which digit is in: the hundreds place? _____

   the ten millions place? _____

   (b) Give the place value and meaning of each digit.

4	tens place	40
1	_____	_____
7	_____	_____

4. Write in standard notation

   (a) 9000 + 50 + 7 _____

   (b) 80 000 + 3000 + 40 _____

5. Write in expanded notation

   (a) 7206 _____

   (b) 504 910 _____

6. Write the number that is

   (a) ten more than 6798 _____

   (b) one hundred less than 8029 _____

Name _____

ESTIMATING PRODUCTS

Worksheet #2: Place Value (Decimals)

1. Write each number as it is read to you.

   (a) _____ (0.3) (b) _____ (0.042)

   (c) _____ (306.08) (d) _____ (3 000.006)

2. Write in words each number.

   (a) 0.009 _____

   (b) 6.53 _____

3. For the number 2 374.5691

   (a) Which digit is in: the hundredths place? _____

   the hundreds place? _____

   (b) Give the place value and meaning of each digit.

5	tenths place	0.5
9	_____	_____
1	_____	_____

4. Write in standard notation

   (a) 0.3 + 0.07 + 0.009 _____

   (b) 70 + 0.06 + 0.0008 _____

5. Write in expanded notation

   (a) 0.908 _____

   (b) 300.040 _____

6. Write the number that is

   (a) one tenth more than 62.94 _____

   (b) one hundredth less than 173.608 _____

Name

ESTIMATING PRODUCTS

Worksheet #3: Order and Comparison

1. Fill in the blanks to extend the number pattern.

   (a) 63, 73, 83, ____, ____

   (b) _____, 3 429, 3529, _____

   (c) _____, 767, 867, 967, ____

   (d) 0.7, 0.8, 0.9, ____, ____

   (e) _____, 0.104, 0.114, _____

   (f) _____, _____, 1.2, 1.5, 1.8

   (g) 9.97, 9.98, 9.99, _____, _____

   (h) _____, _____, 0.1, 0.125, 0.15

2. Circle the numbers equal to the first number in the list.

   (a) 3 3.0 0.3 30 3.00

   (b) 0.07 0.007 0.070 7.00 0.70

3. Find a number between the two given numbers:

   (a) 72 _____ 73

   (b) 0.64 _____ 0.68

   (c) 0.7 _____ 0.74

   (d) 0.03 _____ 0.04

   (e) 2.5 _____ 2.51

   (f) 99.9 _____ 100

4. Circle the greater number

   (a) 41 200 41 198

   (b) 2.357 2.735

   (c) 0.83 0.8

   (d) 0.49 0.5

   (e) 0.6 0.598

   (f) 99.9 99.099

   (g) 547.825 546.925

   (h) 28.201 28.210

   (i) 0.043 0.051

   (j) 0.007 0.0062

   (k) 0.006 0.060

   (l) 0.2 0.199

   (m) 7.007 7.0070

5. Write in order from least to greatest.

   (a) 0.304 0.340 0.043 _____ _____ _____

   (b) 3.22110 3.2011 3.2101 _____ _____ _____

   (c) 4.07 4.0701 4.071 _____ _____ _____

Name

ESTIMATING PRODUCTS

Worksheet #4: Approximate and Rounded Numbers

1. Write an approximate value for each number by writing the first non-zero digit and using zeros to maintain place value. Read the resulting number.

   Examples: 8 672 → 8 000 (eight thousand)

   0.625 → 0.6 (six tenths)

(a)	72	_____	(f)	0.0832	_____
(b)	439	_____	(g)	0.5016	_____
(c)	1 647	_____	(h)	0.00132	_____
(d)	72 643 718	_____	(i)	2.0975	_____
(e)	609 400 005	_____	(j)	634.42	_____

2. Round each number to a number with only a single non-zero digit.

   Examples: 8 672 — between 8 000 and 9 000

   — closer to 9 000 (nine thousand)

   0.625 — between 0.6 and 0.7

   — closer to 0.6 (six tenths)

(a)	68	_____	(i)	0.984	_____
(b)	341	_____	(j)	0.000192	_____
(c)	7 545	_____	(k)	8.702	_____
(d)	9 672	_____	(l)	936.501	_____
(e)	54 987 342	_____	(m)	19.92	_____
(f)	0.0782	_____	(n)	36 342.841	_____
(g)	0.00349	_____	(o)	849.999.99	_____
(h)	0.095	_____	(p)	963 728.046	_____

Name _____

ESTIMATING PRODUCTS

Worksheet #5: Multiplying by Tens

1. (a) 10 x 6 = _____
   (b) 72 x 10 = _____
   (c) 10 x 6.45 = _____
   (d) 0.041 x 10 = _____
   (e) 10 x 0.003 = _____
   (f) 7 x 100 = _____
   (g) 100 x 16 = _____
   (h) 18.2 x 100 = _____
   (i) 100 x 0.495 = _____
   (j) 0.0003 x 100 = _____
   (k) 1 000 x 3 = _____
   (l) 3.4 x 1 000 = _____
   (m) 1 000 x 0.08 = _____
   (n) 0.00419 x 1 000 = _____
   (o) 1 000 x 23.4289 = _____
   (p) 10 000 x 0.0041 = _____

2. (a) 7 x 0.1 = _____
   (b) 0.1 x 83 = _____
   (c) 94.2 x 0.1 = _____
   (d) 0.1 x 0.6 = _____
   (e) 0.073 x 0.1 = _____
   (f) 0.01 x 8 = _____
   (g) 439 x 0.01 = _____
   (h) 0.01 x 3.1 = _____
   (i) 0.5 x 0.01 = _____
   (j) 0.01 x 0.096 = _____
   (k) 2 x 0.001 = _____
   (l) 0.001 x 3284 = _____
   (m) 56.9 x 0.001 = _____
   (n) 0.001 x 0.4 = _____
   (o) 0.038 x 0.001 = _____
   (p) 0.0001 x 237.89 = _____

3. (a) 10 x 100 = _____
   (b) 100 x 1 000 = _____
   (c) 0.1 x 0.1 = _____
   (d) 0.01 x 0.001 = _____
   (e) 0.1 x 100 = _____
   (f) 0.01 x 10 = _____
   (g) 1 000 x 0.01 = _____
   (h) 0.01 x 100 = _____
   (i) 10 x 0.001 = _____
   (j) 0.001 x 10 000 = _____

Name

ESTIMATING PRODUCTS

Worksheet #6: Products of Multiples of Ten

1. (a) 3 x 20 = _____
   (b) 40 x 60 = _____
   (c) 600 x 70 = _____
   (d) 3 x 500 = _____
   (e) 200 x 400 = _____
   (f) 800 x 9 000 = _____
   (g) 7 x 8 000 = _____
   (h) 3 000 x 9 000 = _____

2. (a) 0.2 x 0.7 = _____
   (b) 0.05 x 0.09 = _____
   (c) 0.8 x 0.003 = _____
   (d) 0.04 x 0.0007 = _____
   (e) 0.06 x 0.5 = _____
   (f) 0.03 x 0.004 = _____

3. (a) 3 x 0.7 = _____
   (b) 0.9 x 40 = _____
   (c) 800 x 0.2 = _____
   (d) 0.3 x 500 = _____
   (e) 9 000 x 0.06 = _____
   (f) 0.005 x 3 = _____
   (g) 30 x 0.006 = _____
   (h) 0.09 x 20 = _____
   (i) 8 x 0.04 = _____
   (j) 7 000 x 0.008 = _____

Name

ESTIMATING PRODUCTS

Worksheet #7: Leading Digit Estimators

1. Use leading digit approximations for each factor to estimate the product.

   Example: 374.2 x 0.086 $\quad$ 300 x 0.08 = 24

   (a) 47.8 x 109.2 _____

   (b) 6875.9 x 2.68 _____

   (c) 0.12 x 0.36 _____

   (d) 0.095 x 0.024 _____

   (e) 7.42 x 0.98 _____

   (f) 506.3 x 0.073 _____

2. Use rounded leading digit approximations for each factor to estimate the product.

   Example: 374.2 x 0.086 $\quad$ 400 x 0.09 = 36

   (a) 32.6 x 69.8 _____

   (b) 998.3 x 6.478 _____

   (c) 0.042 x 0.89 _____

   (d) 0.0017 x 0.065 _____

   (e) 8.32 x 0.0049 _____

   (f) 0.00291 x 58.92 _____

3. Use approximations (with one non-zero digit) for the factors which will give the "best" estimate for the product.

   (a) 673.1 x 85.09 _____

   (b) 0.046 x 0.064 _____

   (c) 9785 x 0.066 _____

   (d) 652 x 0.96 _____

Name

ESTIMATING PRODUCTS

Worksheet #8: Fraction Estimators

1. Write the fraction for each decimal.

   (a) 0.5 = _____ (c) 1.5 = _____

   (b) 0.25 = _____ (d) 0.33 $\approx$ _____

2. Find each product.

   (a) $\frac{1}{2}$ x 8 000 = _____ (d) $\frac{1}{3}$ x 0.69 = _____

   (b) $\frac{1}{4}$ x 120 = _____ (e) $\frac{1}{2}$ x 0.072 = _____

   (c) $1\frac{1}{2}$ x 200 = _____

3. Find a common fraction close to each decimal.

   (a) 0.31 _____ (d) 0.49 _____

   (b) 1.52 _____ (e) 0.96 _____

   (c) 0.26 _____

4. Use a fraction to estimate each product.

   Example: 0.49 x 628.9 $\frac{1}{2}$ x 300 = 150

   (a) 1.52 x 79.83 _____

   (b) 0.342 x 1537 _____

   (c) 0.51 x .068 _____

   (d) 0.24 x 2.39 _____

   (e) 0.984 x 0.0062 _____

Name

ESTIMATING PRODUCTS

Worksheet #9: Recognizing Incorrect Answers

1. Estimate each product. Write U if the answer given is unreasonable.

		estimate	
(a)	68 x 723 = 4964	49 000	U
(b)	3143 x 6 = 188558		
(c)	0.32 x 0.59 = 0.01888		
(d)	0.48 x 0.062 = 0.02976		
(e)	3172 x 0.082 = 26.104		
(f)	3.45 x 0.12 = 0.414		

2. State whether the actual answer will be higher or lower than the estimate provided.

		estimate	actual answer
(a)	62 x 73	4200	higher
(b)	59 x 68	4200	
(c)	63 x 69	4200	
(d)	10.1 x 72.34	723.4	
(e)	6.43 x 99.8	643	
(f)	5218 x 0.042	200	

3. Place an X by answers in which the final digit is incorrect.

(a) 374 x 83 = 31 047 ____ (d) 6.04 x 3.9 = 297.98 ____

(b) 0.23 x 4.6 = 1.054 ____ (e) 52.6 x 7.5 = 394.5 ____

(c) 73.8 x 0.091 = 6.7158 ____ (f) 8.3 x 40 = 332 ____

Name

ESTIMATING PRODUCTS

Worksheet #10: Checking Answers

1. Use estimation to place the decimal point in the answer.

	estimate	answer
(a) 74.2 x 63.8	_____	4 7 7 3 3 9 6
(b) 6.2 x 3.75	_____	2325
(c) 8214 x 0.25	_____	2 0 5 3 5
(d) 72.6 x 0.95	_____	6897
(e) 0.35 x 0.62	_____	217
(e) 0.08 x 3240	_____	2592

2. Some of the answers given are wrong. First estimate each product. Then identify incorrect answers giving one of the following reasons:

Unreasonable or misplaced decimal point
Close but should be higher (or lower)
Final digit is wrong

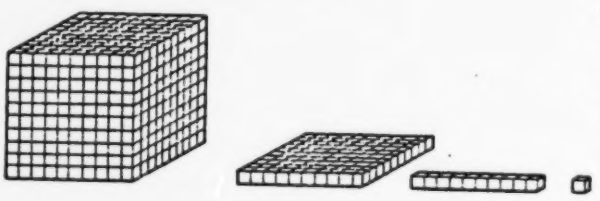
	estimate	answer wrong
(a) 37.8 x 68.9 = 2604.41	_____	_____
(b) 46 x 8.5 = 391	_____	_____
(c) 28.2 x 76 = 243.2	_____	_____
(d) 5.2 x 3.8 = 197.6	_____	_____
(e) 68 x 89.4 = 6479.2	_____	_____
(f) 8.5 x 72.6 = 61.71	_____	_____
(g) 0.037 x 101 = 3.037	_____	_____
(h) 6218 x 0.87 = 54 009.66	_____	_____
(i) .027 x 0.64 = 0.01726	_____	_____
(j) 7200 x 0.25 = 1800	_____	_____
(k) 9.7 x 86.4 = 868.08	_____	_____
(l) 72 x 0.83 = 54.76	_____	_____

Materials and Aids for Teaching Place Value

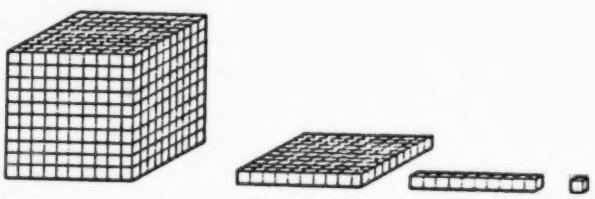
Whole Numbers

Decimal Fractions

1. Base ten blocks

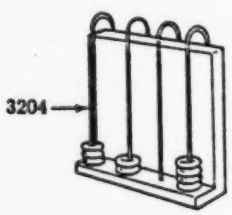


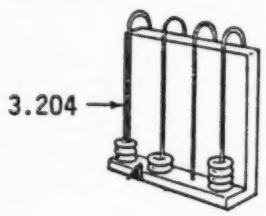
thousand hundred ten one



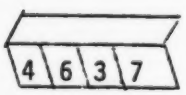
one tenth hundredth thousandth

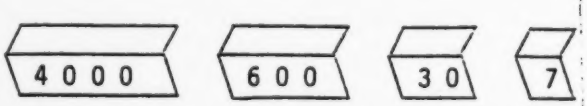
2. Abacus

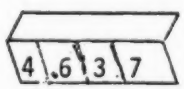


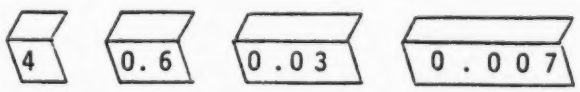


3. Place Value Cards









4. Calculator

(a) Enter 37 482 165

"Wipe out" the 6 in one step leaving the other digits.

(b) Enter the digits from right to left.

40 607 392

(a) Enter 5374.8216

"Wipe out" the 2

(b) Enter the digits from right to left

8300.7056

Place Value

Whole Numbers

billions			millions			thousands					
ındreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
6	8	3	7	0	4	9	٢	0	2	5	7

683 704 910 257

Decimals

	thousands	hundreds	tens	ones	tenths	hundredths	thousandths	
	6	9	4	3	7	2	5	

6943.725

ORDER AND COMPARISON

ORAL ACTIVITIES:

- What number comes before or after a particular number.
  e.g. _____,779,_____ _____ 4999,_____
- "What's my number?" I'm thinking of six hundreds, nine tens, eight ones. What's my number? or I'm thinking of a number one more than 1099. What's my number?
- Teacher starts chain, e.g. 4038 then points to child who gives the next number, e.g. 4039. He in turn points to another child who says 4040. Numbers could progress in 1's, 10's, 100's, or 1000's.

BLACKBOARD ACTIVITIES:

- Write numbers on the board and have the children arrange them from least to greatest, or vice versa.
- Place six or eight cards e.g. 4070 randomly on the chalk ledge. Children come up and rearrange in order.
- Write a three or four-digit number on the blackboard. e.g. 3086. Using each digit once, what different numbers can be made? (3068, 3806, 6830, 8360, etc.) When all possibilities are on the board, call upon children to circle the greatest number, underline the least number, box the odd numbers, order the numbers from greatest to least, etc.
- "Guess My Number?" One child writes a number on a piece of paper between 10 and 100 (or 100 and 1000.) Children try to guess the number in as few tries as possible by asking questions such as, "Is it greater than 50?" "Is it less than 75?" "Does it have a zero in the one's place?" Record 'guesses' on the blackboard.

Place Value: Victoria School District #61, 1982, p. 5-6.

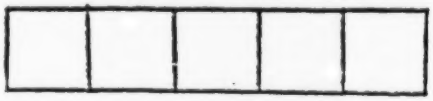
PLACE VALUE GAME

SKILL: Place Value

MATERIALS: Paper, die with numerals 0,1,2,3,4,5, or 1,2,3,4,5,6

PROCEDURE:

1) Let all youngsters make a box like this:



2) Roll the die. The player may put that value in any box he wishes, but the idea of the game is to make the greatest five-digit numeral.

3) Roll the die five times.

4) Example - If the following faces had turned up (0,2,5,3,1) in order, the winner would be the youngster who had 53220. Some youngsters, through choice, would have obtained 25230.

VARIATIONS:

1) Play for the largest and smallest five-digit numeral. In the above example, the lowest numeral would be 02235.

2) Increase number of boxes to six or seven.

3) DECIMALS

Have students make a box with six sections.

Place a decimal between the third and fourth box.

Roll the die. The player may put that value in any box, but as above, play for the largest and smallest decimal number.

Place Value: Victoria School District #61, 1982, p. 14.

IDEAS

Name _____

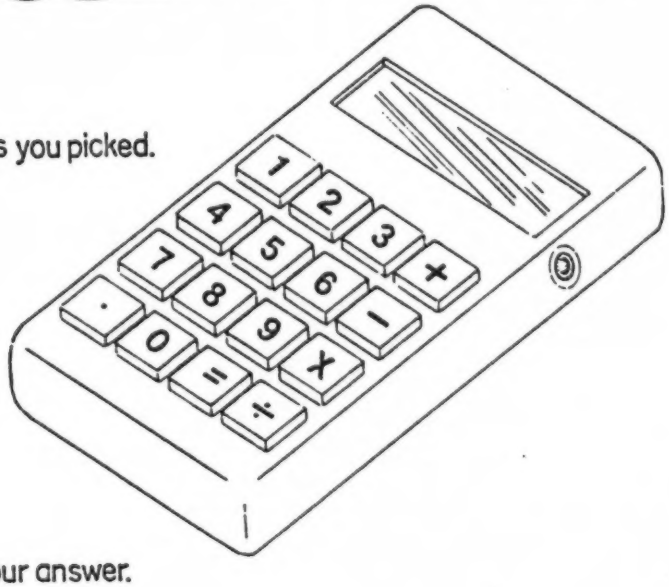
PLAY ESTIMATING DECIMAL PRODUCTS

Pick two numbers.

Mark them with an X.

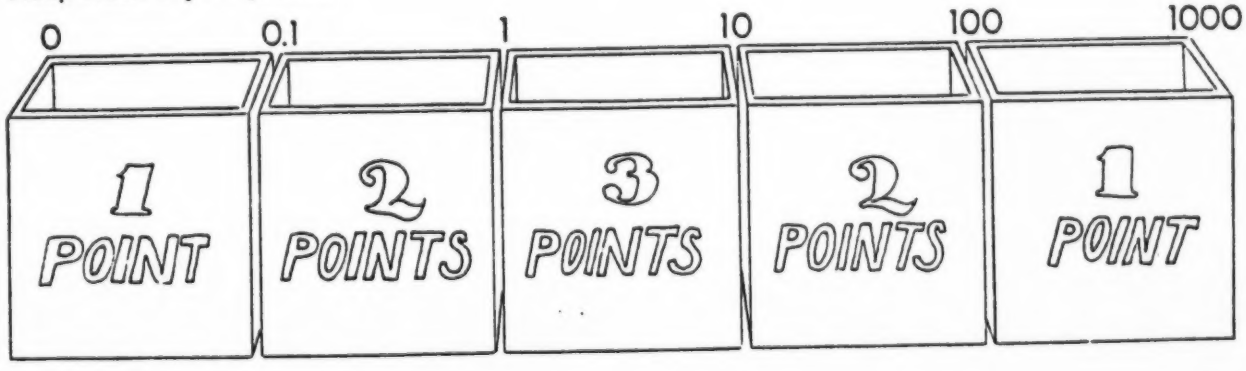
3.6	0.03	13.1	29.6	11.9	0.7	33.7
0.04	21.9	.125	10.1	0.42	0.07	2.9
0.92	19.5	5.52	23.1	9.6	1.8	

Multiply the numbers you picked.



Find the box for your answer.

Keep track of your points.



NCTM

The Product is Right

Purpose: To provide practice in estimating decimal products.

Materials: A calculator, timer and score card for each group of players.

Directions:

1. Divide the class into groups of three to five students. Appoint one member of each group to act as game "host." The others are the players.

2. The host presents a multiplication exercise to the players. Within a fixed period of time (say 30 seconds) each player must write down his or her estimate for the product. The host uses the calculator to find the answer. The round is scored and the host records the points on the score card.

3. Scoring (for four players): Points are awarded for "reasonable" answers. No points are allowed for unreasonable estimates.

Closest estimate	4 points
2nd closest estimate	3 points
3rd closest estimate	2 points
4th closest estimate	1 point

Sample Score Card

		Ann	Bob	Cindy	Dick
1.	16 x 74.3				
2.	237 x 56				
3.	8.6 x 9.82				
4.	489 x 0.51				
5.	0.82 x 0.011				
6.	630 x 0.33				
7.	997.4 x .018				
8.	0.23 x 0.98				
9.	0.71 x 3 247				
10.	0.064 x 149				
	Total				